

Date of completion	7 th of February 2008	Group number	MIF Full time
Instructor	Dr. Paulo M.D.C. Parente	Academic year	2007/2008
Course name	Financial Econometrics-II	Semester, block	First

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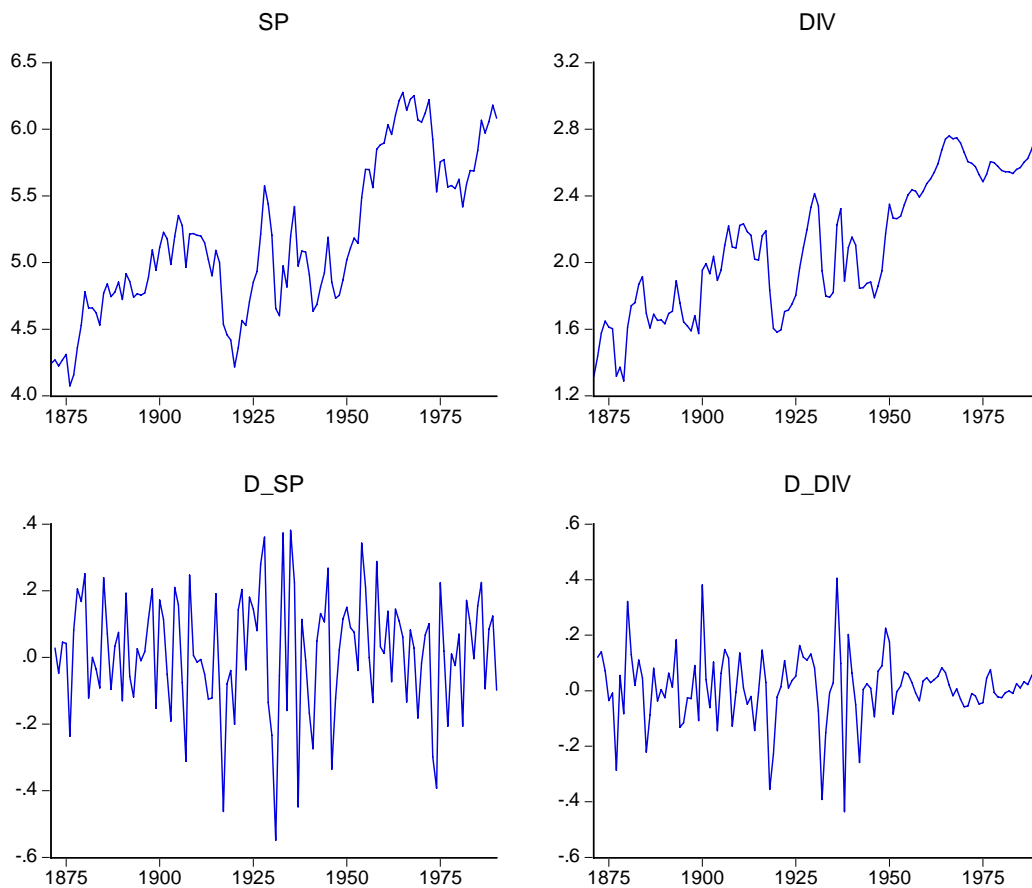
Assignment # 1

Cointegration

Question 1: Step one: stationary or non-stationary?

a. SP and DIV times series

First of all, we would like to check the graph for series of sp and div, which we study. As one can see, sp and div are clearly non-stationary. However, first (log) difference of the series looks like a typical return pattern: fluctuation around zero without any particular trend. This graph suggests there that both series are I(1).



After informal graphic analysis we use ADF¹ test with intercept to test whether y is a stationary time series.

We do not include trend, as it is not theoretically meaningful: there is no reason for an assumption that returns or dividend change has a trend.

ADF test is using the following regression:

$$\Delta y_t = \alpha \cdot y_{t-1} + \beta_1 \cdot \Delta y_{t-1} + \dots + \beta_p \cdot \Delta y_{t-p} + v_t$$

$$H_0 : \alpha = 0$$

$$H_1 : \alpha < 0$$

The null hypotheses — time series is non-stationary (unit root). The p lag is chosen automatically by Eviews with the usage of Schwarz information criterion. The results are presented in table below.

According to the tables both process sp and div series are non-stationary processes: calculated t-stat of ADF test is higher than the critical value of ADF test statistic at 5% level, which is -2.89. This implies that we cannot reject the null hypothesis that there is a unit root (a series is a non-stationary process) at 5% significance level.

Null Hypothesis: DIV has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.840679	0.3594
Test critical values:		
1% level	-3.486064	
5% level	-2.885863	
10% level	-2.579818	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: SP has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.712757	0.4223
Test critical values:		
1% level	-3.486064	
5% level	-2.885863	
10% level	-2.579818	

*MacKinnon (1996) one-sided p-values.

¹ We do not include trend in ADF test as we can hardly give a rationale for it given a graph of y, which clearly shows that there is no trend.

If we look at first differences we will find out that they are stationary processes. According to the tables below both d_{sp} and d_{div} series are stationary processes: calculated t-stat of ADF test is lower than the critical value of ADF test statistic at 5% level, which is -2.89. This implies that we reject the null hypothesis that there is a unit root (a series is a non-stationary process) at 5% significance level.

Null Hypothesis: D_DIV has a unit root
Exogenous: Constant

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-8.469266	0.0000
Test critical values:		
1% level	-3.487046	
5% level	-2.886290	
10% level	-2.580046	

*MacKinnon (1996) one-sided p-values.

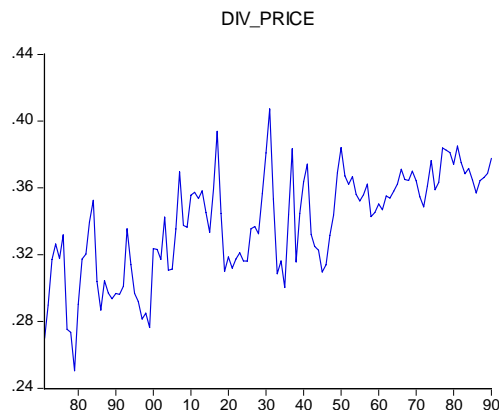
Null Hypothesis: D_SP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.310148	0.0000
Test critical values:		
1% level	-3.487046	
5% level	-2.886290	
10% level	-2.580046	

*MacKinnon (1996) one-sided p-values.

b. DIV/PRICE ratio

According to the graph it is unclear whether series is stationary or not.



According to ADF test we reject null hypothesis of unit root. In this test we include trend as it is theoretically possible meaningful — companies tend to pay more dividends with time compared with price. According to the table below log of div/price is a stationary processes: calculated t-stat of ADF test is lower than the critical value of ADF test statistic at 5% level, which is -3.45. This implies that we reject the null hypothesis that there is a unit root (a series is a non-stationary process) at 5% significance level. Number of lags is chosen automatically by Eviews with the usage of Schwarz information criterion.

Null Hypothesis: DIV_PRICE has a unit root
 Exogenous: Constant, Linear Trend
 Lag Length: 1 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-6.356105	0.0000
Test critical values:		
1% level	-4.037668	
5% level	-3.448348	
10% level	-3.149326	

*MacKinnon (1996) one-sided p-values.

Question 2: Johansen procedure

First we need to estimate the VAR model and test with SC for the optimal number of lags. The optimal number of lags is determined by SC, SP and DIV as endogenous variables in a VAR with a constant. We include only constant, because otherwise it will be a too strong assumption. We do not include trend as it is not theoretically possible. The lag structure given below indicates there that the optimal number of lags – 1.

Lag	LogL	LR	FPE	AIC	SC	HQ
0	111.4964	NA	0.000494	-1.937989	-1.889717	-1.918401
1	129.7766	35.58973	0.000383	-2.190736	-2.045918*	-2.131970
2	135.8103	11.53360	0.000370	-2.226732	-1.985370	-2.128790
3	142.4656	12.48598*	0.000353*	-2.273727*	-1.935821	-2.136609*
4	143.4594	1.829272	0.000372	-2.220520	-1.786069	-2.044224
5	144.6723	2.189617	0.000391	-2.171191	-1.640195	-1.955718
6	146.8000	3.765837	0.000405	-2.138052	-1.510512	-1.883403

In the table below one can find the results of Johansen cointegration test using Unrestricted Co-integration Rank Test (Trace method). As there are only two variables SP and DIV, so there can be only two options: either one or no co-integrating relationship. We reject the null hypothesis of no cointegration vectors since the p-value of 0.04% what is 5% level (significance level). At second step we do not reject the null hypothesis that there is one cointegrating vector since p-value is 13.46% higher than critical 5%.

Date: 02/08/08 Time: 11:39
 Sample (adjusted): 1873 1990
 Included observations: 118 after adjustments
 Trend assumption: Linear deterministic trend
 Series: SP DIV
 Lags interval (in first differences): 1 to 1

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.198076	28.28629	15.49471	0.0004
At most 1	0.018794	2.238749	3.841466	0.1346

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Question 3: VEC

Vector Error Correction Model for SP and DIV with one lag and intercept (no trend) is presented (estimates) in a table below. As one can find cointegrating vector is (1, -1.46, -2.11)'

Vector Error Correction Estimates

Date: 02/08/08 Time: 11:34

Sample (adjusted): 1873 1990

Included observations: 118 after adjustments

Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1	
SP(-1)	1.000000	
DIV(-1)	-1.458353 (0.10629) [-13.7208]	
C	-2.106474	
Error Correction:	D(SP)	D(DIV)
CointEq1	-0.068003 (0.07533) [-0.90270]	0.211442 (0.04181) [5.05758]
D(SP(-1))	0.093503 (0.11361) [0.82301]	0.159539 (0.06305) [2.53034]
D(DIV(-1))	-0.267099 (0.13213) [-2.02145]	0.126212 (0.07333) [1.72118]
C	0.017081 (0.01641) [1.04083]	0.007323 (0.00911) [0.80409]

Testing restriction.

We test the hypothesis that cointegrating vector is $(1,-1)'$. The result of the test is presented below. As the p-value of Chi-square statistics (test) is 0.0472%, which is less than 5% (critical level) we do reject the null hypothesis that cointegrating vector is $(1,-1)'$.

This result is a contradiction with (1), where we have shown that price-to-dividend ratio is stationary. As we do not know the power of tests we can not explain this.

Vector Error Correction Estimates
 Date: 02/08/08 Time: 11:38
 Sample (adjusted): 1873 1990
 Included observations: 118 after adjustments
 Standard errors in () & t-statistics in []

Cointegration Restrictions:
 B(1,1)=1, B(1,2)=-1
 Convergence achieved after 1 iterations.
 Restrictions identify all cointegrating vectors
 LR test for binding restrictions (rank = 1):
 Chi-square(1) 12.22208
 Probability 0.000472

Cointegrating Eq:	CointEq1
SP(-1)	1.000000
DIV(-1)	-1.000000
C	-3.067566

Question 4

Part 1. Stationarity

SP. First, we perform unit root (ADF) in levels with intercept but without trend for SP. As it is shown in table below, SP is not a stationary time series. P-value is higher than 5% so we do not reject the null hypothesis that SP is not stationary.

Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.032611	0.7403
Test critical values:		
1% level	-3.480818	
5% level	-2.883579	
10% level	-2.578601	

DIV. We perform unit root in levels with intercept but without trend for DIV. As it is shown in table below, DIV is not a stationary time series. P-value is higher than 5% so we do not reject the null hypothesis that DIV is not stationary.

Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

		t-Statistic	Prob.*
<hr/>			
Augmented Dickey-Fuller test statistic		-1.895682	0.3335
Test critical values:	1% level	-3.480818	
	5% level	-2.883579	
	10% level	-2.578601	

SP/DIV. We perform unit root in levels with intercept but without trend for SP/DIV. As it is shown in table below, SP/DIV is a stationary time series. P-value is lower than 5% so we reject the null hypothesis that SP/DIV is not stationary.

		t-Statistic	Prob.*
<hr/>			
Augmented Dickey-Fuller test statistic		-4.019006	0.0019
Test critical values:	1% level	-3.486064	
	5% level	-2.885863	
	10% level	-2.579818	

Part 2. Johansen Procedure

First we determine with SC criteria and VAR model as in (2) optimal number of lags, which is 1 (See Table below).

Included observations: 113

Lag	LogL	LR	FPE	AIC	SC	HQ
0	111.4964	NA	0.000494	-1.937989	-1.889717	-1.918401
1	129.7766	35.58973	0.000383	-2.190736	-2.045918*	-2.131970
2	135.8103	11.53360	0.000370	-2.226732	-1.985370	-2.128790
3	142.4656	12.48598*	0.000353*	-2.273727*	-1.935821	-2.136609*
4	143.4594	1.829272	0.000372	-2.220520	-1.786069	-2.044224
5	144.6723	2.189617	0.000391	-2.171191	-1.640195	-1.955718
6	146.8000	3.765837	0.000405	-2.138052	-1.510512	-1.883403

In the table below one can find the results of Johansen Co-integration Test using Unrestricted Co-integration Rank Test (Trace method). As there are only two variables

SP and DIV, so there can be only two options: either one or no co-integrating relationship. We reject the null hypothesis of no cointegration vectors since the p-value of 1.87% is lower than 5% level (significance level). At second step we do not reject the null hypothesis that there is one cointegrating vector since p-value is 25.12% higher than critical 5%.

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.122193	18.25960	15.49471	0.0187
At most 1	0.010078	1.316841	3.841466	0.2512

Part 3

Vector Error Correction Model for SP and Div using one lag variable is estimated using intercept (no trend) in cointegration equation and VAR. The estimates of cointegrating vector has change to (1, -1.82, -1.37)'

Standard errors in () & t-statistics in []

Cointegrating Eq:	CointEq1	
SP(-1)	1.000000	
DIV(-1)	-1.822367 (0.16910) [-10.7769]	
C	-1.367058	
Error Correction:	D(SP)	D(DIV)
CointEq1	-0.041097 (0.05192) [-0.79159]	0.116026 (0.02894) [4.00916]
D(SP(-1))	0.110774 (0.10167) [1.08952]	0.213492 (0.05668) [3.76684]
D(DIV(-1))	-0.246837 (0.13263) [-1.86112]	0.135633 (0.07393) [1.83456]
C	0.019815 (0.01567) [1.26455]	0.004033 (0.00873) [0.46168]

Testing restriction

We test the hypothesis that cointegrating vector is $(1,-1)'$. The result of the test is presented below. As the p-value of Chi-square statistics (test) is 0.0346%, which is less than 5% (critical level), so we do reject the null hypothesis that cointegrating vector is $(1,-1)'$.

This result is a contradiction with (1), where we have shown that price-to-dividend ratio is stationary.

Cointegration Restrictions:

$B(1,1)=1, B(1,2)=-1$

Convergence achieved after 1 iterations.

Restrictions identify all cointegrating vectors

LR test for binding restrictions (rank = 1):

Chi-square(1) 12.80289

Probability 0.000346

Cointegrating Eq:	CointEq1
SP(-1)	1.000000
DIV(-1)	-1.000000
C	-3.145315

Question 5: Using Earnings

In the end we find that (for earnings) everything hold except:

- 1) Cointegrating vector for a sample 1871-1990, which does not include dot com bubble, is very close to $(1,-1)$. This might be a prove for a statement that earnings provide better explanation of price than dividends
- 2) For a subsample 1871-1990 there is more evidence of cointegration of prices and earnings: we do not reject the hypothesis of cointegration for a full sample at 10% significance level, while for a subsample we do not reject at 5% level.

First we will perform the analysis for earning for the full sample – 1871-2002. Than we shortly describe results for a subsample 1871-2002.

Part 1. Stationarity

EARNINGS. Using ADF test with constant but without a trend we find that earnings are non-stationary as p-value is higher than 5% (critical level), so we do not reject the null hypothesis that earning are non-stationary.

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.263817	0.1853
Test critical values:		
1% level	-3.480818	
5% level	-2.883579	
10% level	-2.578601	

SP/EARNINGS are stationary as p-value of ADF test is lower than 5%, so we reject the null hypothesis of non-stationarity.

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-5.382118	0.0000
Test critical values:		
1% level	-3.480818	
5% level	-2.883579	
10% level	-2.578601	

Part 2. Johansen procedure

First we determine the optimal number of lags using VAR model with constant but without trend. We find using SC that number of optimal lags -1.

Included observations: 113

Lag	LogL	LR	FPE	AIC	SC	HQ
0	42.57730	NA	0.001672	-0.718182	-0.669910	-0.698594
1	59.37035	32.69443*	0.001333*	-0.944608*	-0.799791*	-0.885843*
2	61.74087	4.531269	0.001372	-0.915768	-0.674406	-0.817826
3	63.11192	2.572236	0.001438	-0.869238	-0.531331	-0.732119
4	68.15726	9.286999	0.001412	-0.887739	-0.453288	-0.711443
5	72.64269	8.097578	0.001401	-0.896331	-0.365335	-0.680858
6	72.98051	0.597922	0.001496	-0.831513	-0.203973	-0.576864

In the table below one can find the results of Johansen Co-integration Test using Unrestricted Co-integration Rank Test (Trace method). As there are only two variables SP and DIV, so there can be only two options: either one or no co-integrating relationship. We reject the null hypothesis of no cointegration vectors at 10% level of significance since the p-value of test is 5.02%: higher p-value here might be a signal of worse cointegration relationship between earnings and prices. At second step we do not reject

the null hypothesis that there is one cointegrating vector since p-value is 18.67% higher than critical 5%.

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.100289	15.48193	15.49471	0.0502
At most 1	0.013320	1.743243	3.841466	0.1867

Part 2. VEC

Vector Error Correction Model for SP and EARNINGS using one lag variable is estimated using intercept (no trend) in cointegration equation and VAR. The estimates of cointegrating vector has change to $(1, -1.4, -1.57)'$, which is different from estimates for DIV.

Cointegrating Eq:	CointEq1	
SP(-1)	1.000000	
EARN(-1)	-1.400482 (0.16852) [-8.31046]	
C	-1.570270	
Error Correction:	D(SP)	D(EARN)
CointEq1	-0.024405 (0.04096) [-0.59581]	0.159986 (0.04603) [3.47607]
D(SP(-1))	0.099467 (0.09802) [1.01480]	0.486298 (0.11013) [4.41548]
D(EARN(-1))	-0.080764 (0.07426) [-1.08762]	-0.038464 (0.08344) [-0.46098]
C	0.018201 (0.01575) [1.15547]	0.002328 (0.01770) [0.13155]

Testing restrictions

We test the hypothesis that cointegrating vector is $(1,-1)'$. The result of the test is presented below. As the p-value of Chi-square statistics (test) is 2.6%, which is less than 5% (critical level), so we do reject the null hypothesis that cointegrating vector is $(1,-1)'$.

This result is a contradiction with a stationarity of sp/earn.

Cointegration Restrictions:	
B(1,1)=1,B(1,2)=-1	
Convergence achieved after 1 iterations.	
Restrictions identify all cointegrating vectors	
LR test for binding restrictions (rank = 1):	
Chi-square(1)	4.937032
Probability	0.026287

Cointegrating Eq:	CointEq1
SP(-1)	1.000000
EARN(-1)	-1.000000
C	-2.639019

Robustness Check, using 1871-1990 data

Stationarity. Results do not change

EARNINGS – not stationary as p-value is higher than 5%

Lag Length: 0 (Automatic based on SIC, MAXLAG=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.246908	0.1911
Test critical values:		
1% level	-3.486064	
5% level	-2.885863	
10% level	-2.579818	

SP/EARNINGS –stationary as p-value is lower than 5%

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-5.102622	0.0000
Test critical values:		
1% level	-3.486064	
5% level	-2.885863	
10% level	-2.579818	

Cointegration test. Results do not change. However, we observe more evidence of cointegration as the first null hypothesis (null: no cointegrating vectors) is rejected at 5% significance level, rather than 10% for the full sample. This may be due to the dot com bubble.

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.117066	17.44368	15.49471	0.0251
At most 1	0.023053	2.752156	3.841466	0.0971

VEC. Great Change. We observe that cointegrating vector is very close to theoretical levels.

Cointegrating Eq:	CointEq1	
SP(-1)	1.000000	
EARN(-1)	-1.085350 (0.13433) [-8.07978]	
C	-2.365409	
Error Correction:	D(SP)	D(EARN)
CointEq1	-0.043438 (0.05704) [-0.76157]	0.217941 (0.06268) [3.47719]
D(SP(-1))	0.086154	0.446836

	(0.10497)	(0.11535)
	[0.82075]	[3.87377]
D(EARN(-1))	-0.125849	-0.050493
	(0.07747)	(0.08513)
	[-1.62451]	[-0.59314]
C	0.015794	0.006645
	(0.01647)	(0.01809)
	[0.95921]	[0.36722]

Testing restrictions

The null hypothesis (cointegrating vector of $(1, -1)$) is not rejected: p-value is higher than 5%. So we believe this is evidence proves that Earnings and Prices are cointegrated with a theoretically meaningful cointegration vector. That does not contradict the finding that price-earnings ratio is stationary.

Cointegration Restrictions:

$$B(1,1)=1, B(1,2)=-1$$

Convergence achieved after 1 iterations.

Restrictions identify all cointegrating vectors

LR test for binding restrictions (rank = 1):

Chi-square(1) 0.315124

Probability 0.574553

Cointegrating Eq:	CointEq1
SP(-1)	1.000000
EARN(-1)	-1.000000
C	-2.585518

Question 6: Long-Horizon regressions

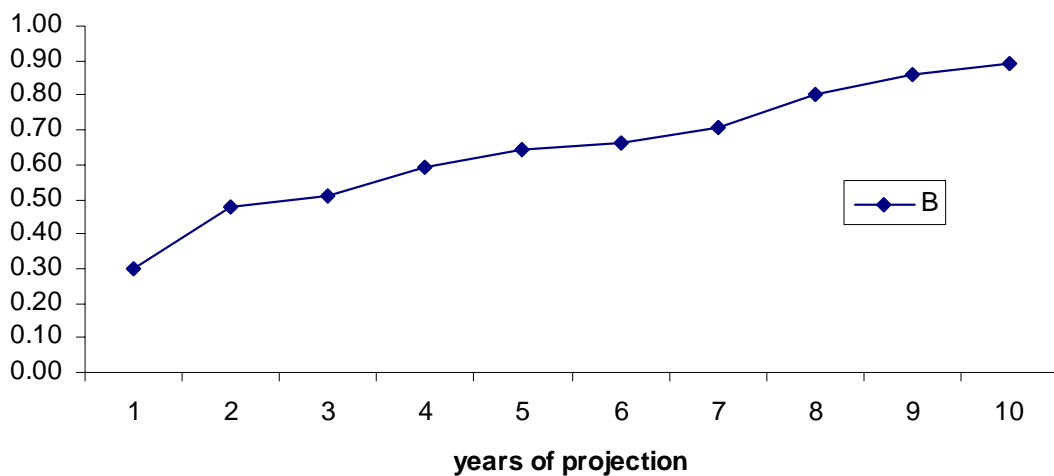
We use the following long-horizon regression to show that price-dividend ratio predict future long-horizon returns.

$$y_{t+h} - y_t = \alpha_h + \beta_h (y_t - x_t) + \varepsilon_{ht},$$

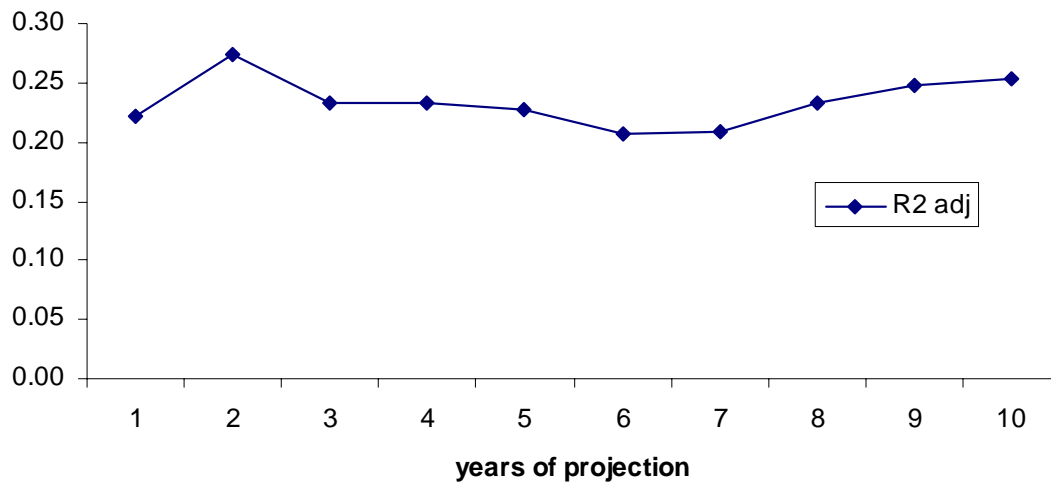
Estimates of regression are presented in the table below. We estimate 10 betas for 10 years of projection – 1, 2, ..., 10. We use Newey-West standard errors (HAC s.e). Data from 1871 to 2002.

As one can find beta (influence of price-to-dividends ratio) is increasing as a function of horizon. R2 adj. on average does not change significantly. Estimates of are significant at 5% level for all regressions, so in each case we reject the null hypothesis that beta is equal to zero (no effect of price-dividend ratio on returns).

B estimates and projection horizon



R2 adj and projection horizon



	1 year		2 years		3 years		4 years		5 years	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
C	-0.91	0.00	-1.44	0.00	-1.52	0.00	-1.76	0.00	-1.91	0.00
Beta	0.30	0.00	0.48	0.00	0.51	0.00	0.59	0.00	0.64	0.00
R-squared	0.23		0.28		0.24		0.24		0.23	
Adj. R2	0.22		0.27		0.23		0.23		0.23	

	6 years		7 years		8 years		9 years		10 years	
	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value	Coefficient	P-value
C	-1.96	0.00	-2.08	0.00	-2.36	0.00	-2.53	0.00	-2.62	0.00
Beta	0.66	0.00	0.71	0.00	0.80	0.00	0.86	0.00	0.89	0.00
R-squared	0.21		0.22		0.24		0.26		0.26	
Adjusted R-squared	0.21		0.21		0.23		0.25		0.25	